XI PHYSICS

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[GEOMETRICAL OPTICS] CHAPTER NO. 10

Lens

A piece of transparent material designed in manner to diverge or converge refracted light is called a lens.

Converging Lens (Convex Lens)

The lens which converge the parallel beam of light passing through it at a certain point (i.e. Principal Focus) is known as Converging lens.

OR

The lens which is thick from the centre and thin from edges is also called convex lens.

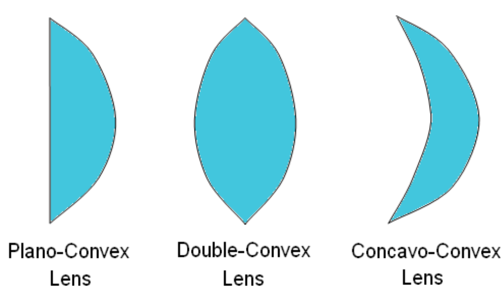
Diverging Lens (Concave Lens)

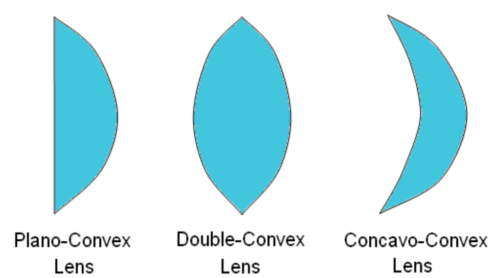
The lens which diverges the parallel beam of light passing through it is known as Diverging Lens or Concave Lens.

OR

The lens which is thin from the centre and thick from edges is also called concave lens.

Other types of Lenses





Ray diagram for Convex Lens

1) If a ray is coming parallel to Principal Axis or coming from infinity it converges at Principal Focus
2) If a ray is striking at Optical Centre it goes through lens un-deflected
3) If a ray is passing through Principal focus of the lens it becomes parallel to Principal Axis after refraction.

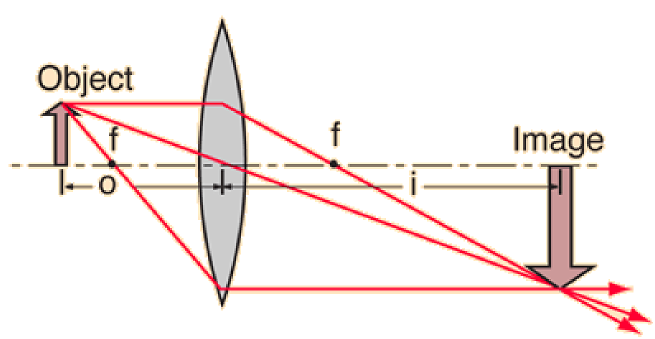
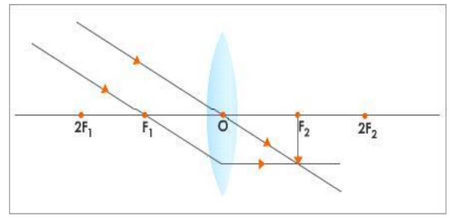


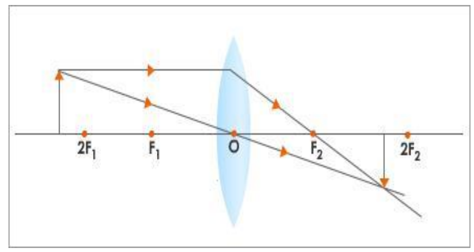
Image formed by Convex Lens

1) When the object is placed at infinity



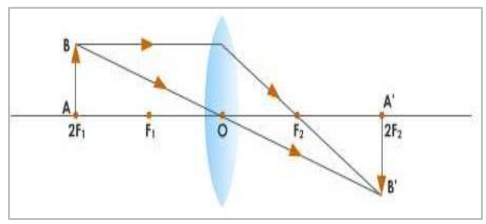
a) The image will form at the principal focus (F).
b) The image will be real and inverted.
c) The image will be very small in size.

2) When the object is placed beyond 2F



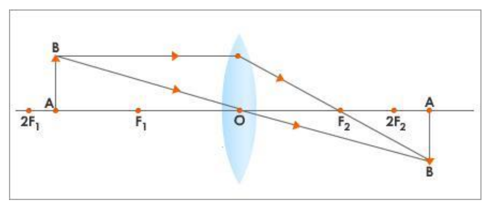
a) The image will form between F and 2F.
b) The image will be real and inverted.
c) The image will be smaller in size.

3) When the object is placed at 2F



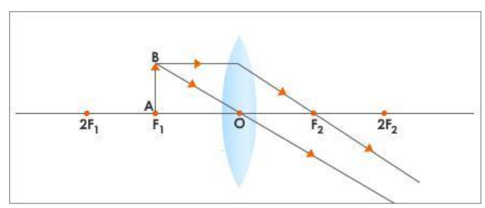
a) The image will form at 2F.
b) The image will be real and inverted.
c) The image will be equal in the size of object.

4) When the object is placed between F and 2F



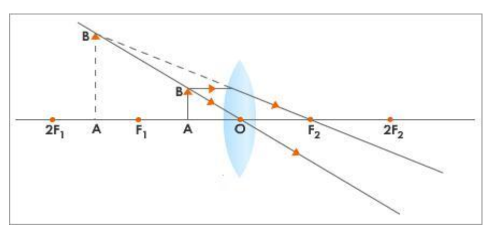
a) The image will form beyond 2F.
b) The image will be real and inverted.
c) The image will be magnified.

5) When the object is placed at F



a) The image will form at infinity.
b) The image will be real and inverted.
c) The image will be highly magnified.

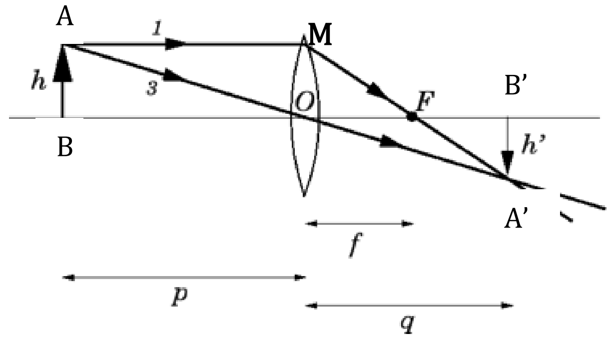
6) When the object is placed between the pole (P) and F



a) The image will form on the same side of object.
b) The image will be virtual and erect.
c) The image will be magnified.

THIN LENS FORMULA

Thin lens formula relates focal length ‘f’ distance of object from lens ‘p’ and distance of image ‘q’ from lens. It can only be applied on thin lenses.



Formula for Convex Lens

Consider an object, which is at a distance ‘p’ from the lens and a real and inverted image, is obtained at a distance ‘q’ from lens as shown in the figure. Since the triangles AOB and A′O′B′ are similar, therefore we may write

$$\frac{A'B'}{AB} = \frac{OB'}{OB}$$

$$\frac{A'B'}{AB} = \frac{q}{p} \dots\dots\dots (1)$$

Also triangles MFO and A′FB′ are similar, therefore we may also write

$$\frac{A'B'}{MO} = \frac{FB'}{FO}$$

$$\frac{A'B'}{AB} = \frac{q-f}{f} \dots\dots\dots (2)$$

Equating (1) and (2)

$$\frac{q}{p} = \frac{q-f}{f}$$

$$\frac{q}{p} = \frac{q}{f} - \frac{f}{f}$$

$$\frac{q}{p} = \frac{q}{f} - 1$$

Dividing by ‘q’ on both sides

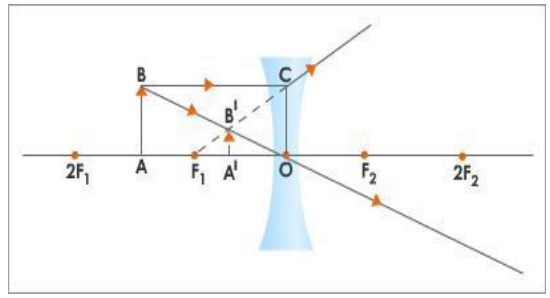
$$\frac{q}{qp} = \frac{q}{qf} - \frac{1}{q}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

The above equation is called Thin Lens Formula

Formula for Concave Lens



Consider a thin concave lens of focal length 'f' and an object is placed at a distance 'p' from it; image is obtained at a distance 'q' from lens. As shown in figure.

Since triangles ABO and A′B′O are similar, so we may write

$$\frac{A'B'}{AB} = \frac{A'O}{AO}$$

$$\frac{A'B'}{AB} = \frac{q}{p} \dots\dots\dots (1)$$

Also, triangles COF and A′B′F are similar, therefore we may also write,

$$\frac{A'B'}{CO} = \frac{A'F}{OF}$$

$$\frac{A'B'}{AB} = \frac{f-q}{f} \dots\dots\dots (2)$$

Equating (1) and (2), we get

$$\frac{q}{p} = \frac{f-q}{f}$$

$$\frac{q}{p} = \frac{f}{f} - \frac{q}{f}$$

$$\frac{q}{p} = 1 - \frac{q}{f}$$

Dividing by 'q' on both sides,

$$\frac{p}{qp} = \frac{1}{q} - \frac{q}{qf}$$

$$\frac{1}{p} = \frac{1}{q} - \frac{1}{f}$$

$$\frac{1}{p} - \frac{1}{q} = -\frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{(-q)} = \frac{1}{(-f)}$$

Since in concave lens, q and f are negative, so we may write
Therefore, by sign convention,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Therefore, same formula is used for concave lens.

Power of Lenses

"The reciprocal of focal length of a lens is termed as power of lens if focal length is taken in metre."

Mathematically,

$$P = \frac{1}{f_{(meters)}}$$

S.I unit of Power is 'dioptre', also known as 'refractive power'.

Power of Combination of Lens,

When two thin lenses (or more) of focal lengths f_1 and f_2 are place in contact with each other, then the power of combination is simply the sum of power of both lenses.

$$P = P_1 + P_2$$

We may also write it like,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Combination of Lenses

Consider a convex lens of focal length 'f' an object is placed in front of lens at a distance 'p' and an image is obtained at a distance 'q_1' from the lens therefore using convex lens equation we have,

$$\frac{1}{f_1} = \frac{1}{p} + \frac{1}{q_1} \ldots\ldots\ldots (1)$$

Now if another convex lens of focal length f_2 is placed with first lens then image at 'B' will acts as an object for this 2nd lens and final image is obtained at a distance 'q' from this second lens therefore,

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{q} \ldots\ldots\ldots (2)$$

Since lenses are very thin therefore, neglecting their separation we can write,

$$p_2 = -q_1 \ldots\ldots\ldots \text{(since image is virtual)}$$

Now using this equation (2) becomes,

$$\frac{1}{f_2} = -\frac{1}{q_1} + \frac{1}{q} \ldots\ldots\ldots (3)$$

Adding equation (1) and (3)

$$(1) \Rightarrow \frac{1}{f_1} = \frac{1}{p} + \frac{1}{q_1}$$

$$(2) \Rightarrow \frac{1}{f_2} = -\frac{1}{q_1} + \frac{1}{q}$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$$

This equation shows that the combination of lenses can be treated as a single lens whose focal length's reciprocal equal to the sum of reciprocals of the individual lenses.

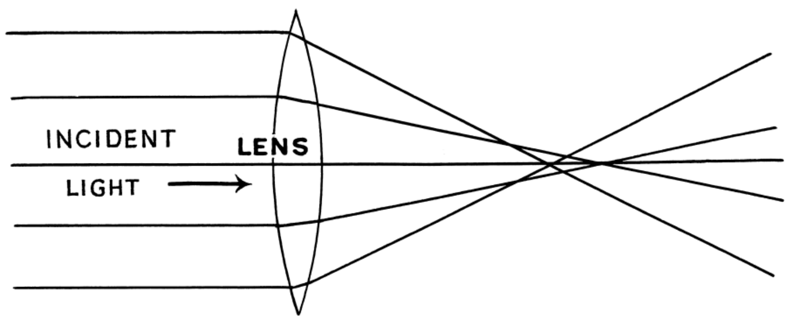
Lens Aberration

"The deviation in shape, position and colour of images produced by the lens in comparison to the calculated ones is called lens aberration".

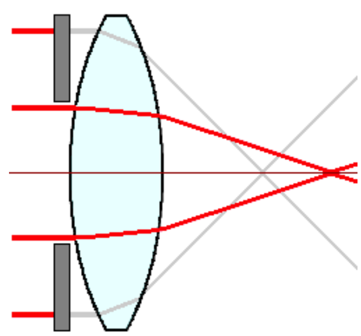
There are two general types of aberration in lenses.

1) Spherical Aberration

"The failure of a lens to form a point image of a point object on its optic axis is called spherical aberration".



Rays refracted from edges are collected before actual focal point while rays from middle part of the lens are collected near to focal points thus we get many images near focal point. This aberration is due to large aperture of lens.

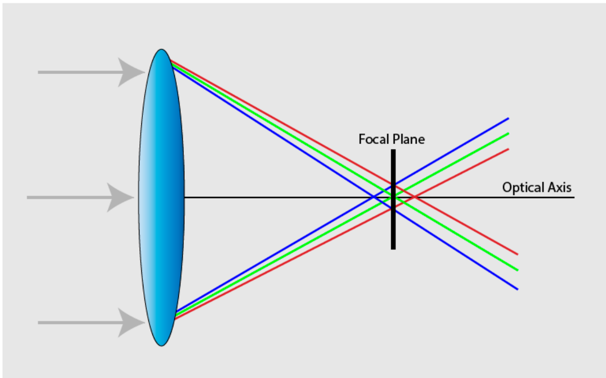


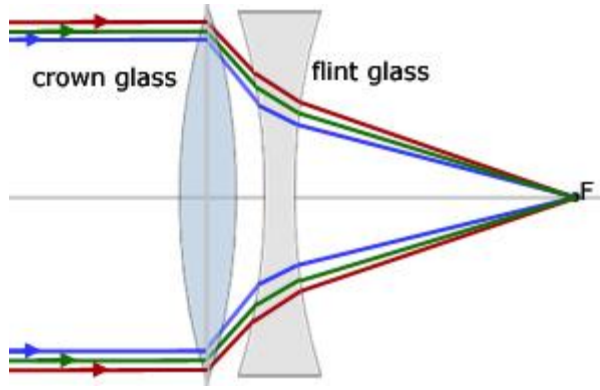
Correction:

This aberration can be reduced by reducing the size of aperture or by just sending light from the middle of the lens.

2) Chromatic Aberration

Chromatic aberration is the distortion of the image due to dispersion of light in the lenses, when white light is used. This aberration is due to the variation of indices with wavelength. Dispassion causes the focal length of a lens to be somewhat different for different wavelengths so different colors are imaged at different points and we get colorful lining images.





Correction:
This aberration can be reduced by using monochromatic light or by combining convex lens with a concave lens.

Least Distance of Distinct Vision (d)

"Minimum distance between eye and object at which object can be seen clearly is called least distance of distinct vision".

Normally this distance is about 25 cm.

Magnification

"The ability of lens to make larger image as compare to object is called magnification".

There are two types of magnification,
1) Linear Magnification
2) Angular Magnification

1) Linear Magnification

"The ratio between the size of image to the size of object is called linear magnification."

$$M = \frac{\text{size of image}}{\text{size of object}}$$

$$M = \frac{h_i}{h_0}$$

Magnification is also equal to,

$$M = \frac{\text{image distance}}{\text{object distance}}$$

$$M = \frac{q}{p}$$

For real images, magnification is taken as positive and for virtual images it is negative.

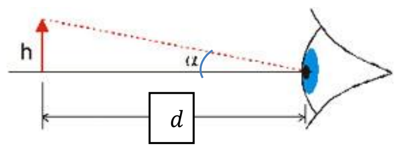
2) **Angular Magnification**

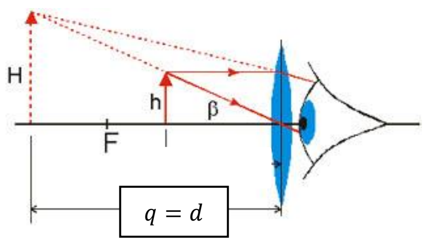
It is the ratio of angle formed by the image using lens and angle formed by object without lens.

$$M = \frac{\beta}{\alpha}$$

Where,

$$\beta = \text{optical angle at eye with lens}$$
$$\alpha = \text{optical angle at eye without lens}$$





When angles are small, then they are nearly equal to their tangents. From figure we find

$$\alpha = \tan\alpha = \frac{\text{size of the object}}{\text{distance of the object}} = \frac{AB}{d}$$

And,

$$\beta = \tan\beta = \frac{\text{size of the image}}{\text{distance of the image}} = \frac{A'B'}{q} = \frac{A'B'}{d}$$

Since,

$$M = \frac{\beta}{\alpha} = \frac{\frac{A'B'}{d}}{\frac{AB}{d}} = \frac{A'B'}{AB} = \frac{\text{height of image}}{\text{height of object}}$$

As we already know that,

$$\frac{\text{height of the image}}{\text{height of the object}} = \frac{\text{distance of image}}{\text{distance of object}} = \frac{q}{p}$$

Therefore,

$$M = \frac{q}{p}, \text{or} = \frac{d}{p}$$

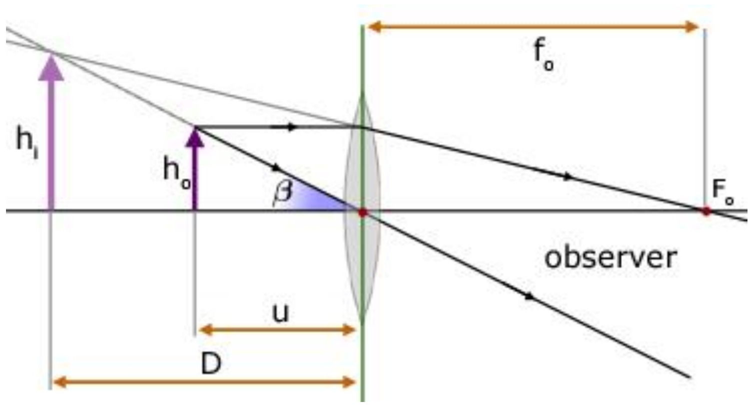
For real images, magnification is taken as positive and for virtual images it is negative.

Magnifying Glass (Or Simple Microscope)

"A convex lens by which a large and clear image of an object, placed at least distance of distinct vision, is obtained is called magnifying glass or simple microscope".

Principle

When an object is placed between a convex lens and its focus an erect virtual and magnified image is formed on the same side as the object. A single convex lens of short focal length thus used as a simple microscope.



Using thin lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Here, $q = -d$

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiplying by 'd' on both sides, we get

$$\frac{d}{f} = \frac{d}{p} - 1$$

$$\frac{d}{f} + 1 = \frac{d}{p}$$

Since $\frac{d}{p} = M$

$$M = \frac{d}{f} + 1$$

Compound Microscope

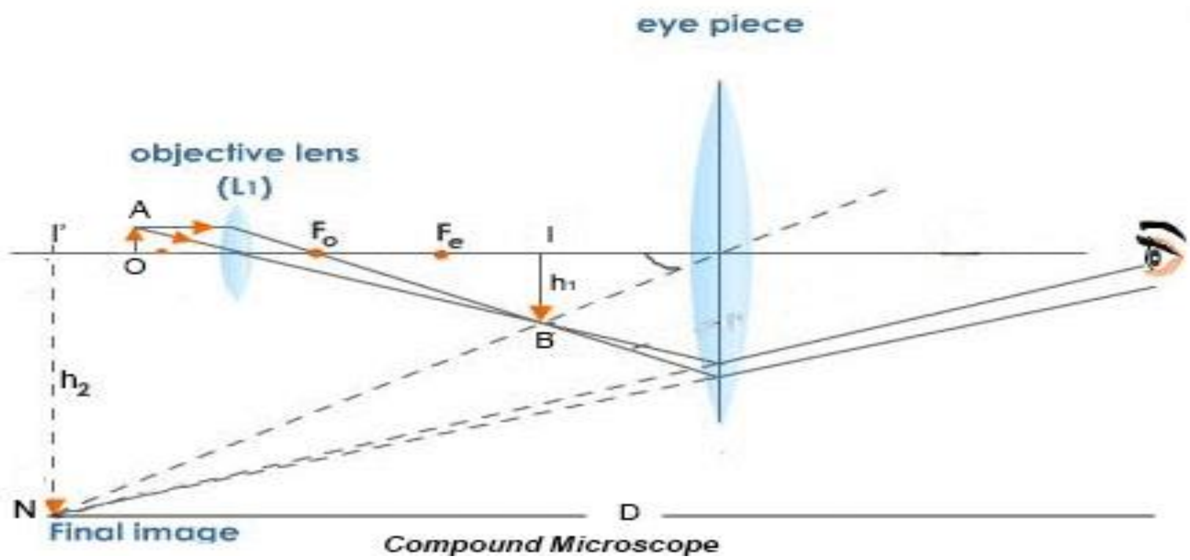
"A compound microscope is an optical instrument, which is used to see small objects with high magnification."

Construction

Usually two convex lenses are used in a compound microscope. Both lenses are fitted on a metal tube. The lens near eye is called eyepiece and other lens is called objective. The focal length of objective 'f_o' is smaller than focal length of eyepiece 'f_e'.

Principle and Working

Compound microscope based on a simple convex lens and a magnifying glass. The objective forms an inverted, virtual image with in the focal length of eyepiece which is used as a magnifying glass and produces a large virtual final image of first image.



MAGNIFICATION

We can define magnification of compound microscope by taking the ratio of height of the final image with the height of starting object.

$$M = \frac{A''B''}{AB}$$

Multiplying and dividing by $A'B'$, we get

$$M = \frac{A''B''}{AB} \times \frac{A'B'}{A'B'}$$

We may write it

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = M_e \times M_o \dots \dots \dots (1)$$

Here,

$$M_o = \frac{q_o}{p_o}$$

And,

$$M_e = \left(1 + \frac{d}{f_e}\right)$$

Since, eyepiece is working as a magnifying glass

Substituting both values in equation (1) we get,

$$M = \left(1 + \frac{d}{f_e}\right)\left(\frac{q_0}{p_o}\right)$$

Or,

$$M = \frac{q_o}{p_o}\left(1 + \frac{d}{f_e}\right) \dots \dots \dots (2)$$

Also, we can approximate this equation,

Since we can see from diagram that the object distance from objective is very near to focal length of objective, therefore we may write,

$$p_o \approx f_o$$

And since the distance between the two lenses is

$$L = q_o + p_e$$

Here since, p_e is very small, if we approximate it to zero then

$$L \approx q_o$$

Then equation (2) can be written as,

$$M = \frac{L}{f_o}\left(1 + \frac{d}{f_e}\right)$$

Telescope

"An optical device, which is used to view the distant objects, is called TELESCOPE."

Astronomical Telescope

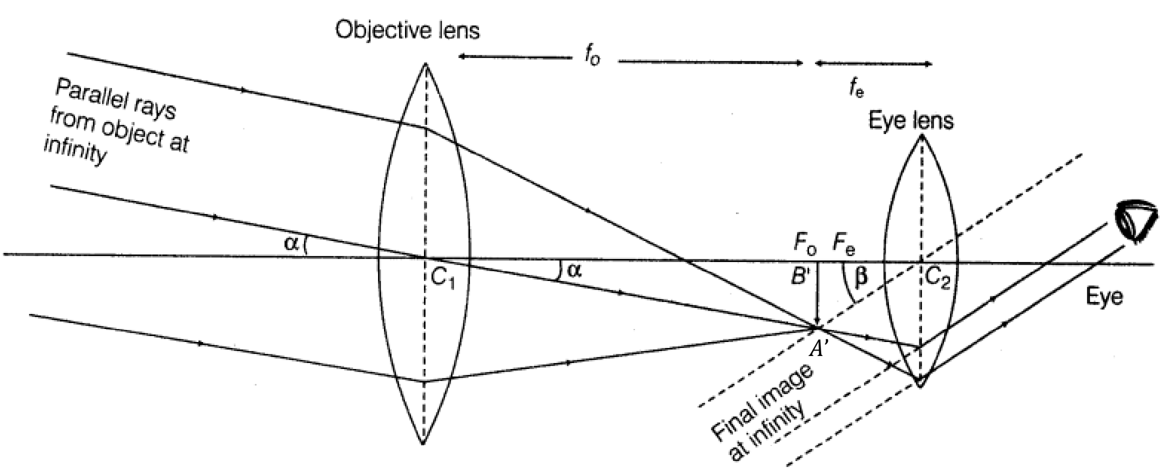
Astronomical telescope is an instrument used to observe heavenly bodies such as moon stars etc.

Construction

It consists of two convex lenses one is objective and other is called eyepiece. Focal length 'f_o' is greater than 'f_e' (focal length of eyepiece) both lenses are fitted on the either end of a metal tube.

Principle and Working

Light rays, coming from distant objects are almost parallel and forms small inverted real image on the focal plane of objective lens. Eyepiece receives this image at its focus and forms a very large final image at infinite distance.



Consider triangle $A'B'C_1$

We may write

$$\tan\alpha = \frac{p}{b}$$
$$\tan\alpha = \frac{A'B'}{B'C_1}$$
$$\tan\alpha = \frac{A'B'}{f_o}$$
Since α is very small, therefore we may write, $\tan\alpha \approx \alpha$
$$\alpha = \frac{A'B'}{f_o} \ldots\ldots\ldots (1)$$
Now consider triangle $A'B'C_2$

We may write
$$\tan\beta = \frac{p}{b}$$
$$\tan\beta = \frac{A'B'}{B'C_2}$$
$$\tan\beta = \frac{A'B'}{f_e}$$
Since β is very small, therefore we may write, $\tan\beta \approx \beta$
$$\beta = \frac{A'B'}{f_e} \ldots\ldots\ldots (2)$$
Since, we know that,
$$M = \frac{\beta}{\alpha}$$
$$M = \frac{A'B'/f_e}{A'B'/f_o}$$
$$M = \frac{1/f_e}{1/f_o}$$
$$M = \frac{f_o}{f_e}$$
Length of telescope,
$$L = f_o + f_e$$

Galilean Telescope
In 1906, Galileo made a telescope for observation of heavenly objects. This telescope forms an erect image of the bodies

Construction
It consists of an objective which is convex lens of large focal length 'f_o' and eyepiece which is a concave lens of short focal length 'f_e'. Both lenses are fitted on a metal tube.

Principle and Working
Objective collects the rays coming from distant object and makes an image on its focus. Eyepiece is placed between the objective and image in such a way that its focus coincide with the focus of objective thus a large and erect image is obtained.

Magnification of Galilean Telescope

$$M = \frac{f_o}{f_e}$$

Length of Galilean Telescope

$$L = f_o - f_e$$

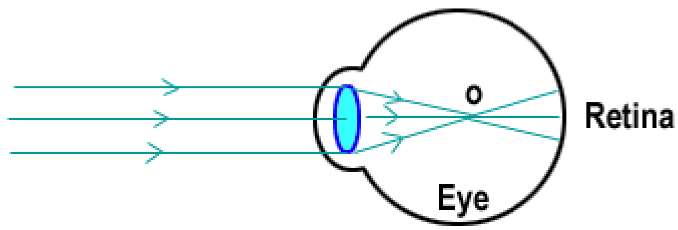
Defects of Vision:

There are four common defects of vision:

1. Short sightedness or myopia
2. Long sightedness or hyper-metropia
3. Astigmatism
4. Presbyopia

Short sightedness or Myopia

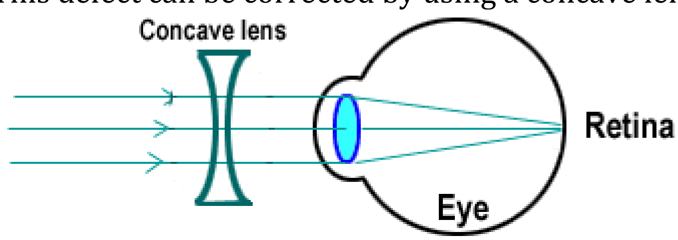
In Myopia, a person cannot see distant objects clearly, but he can see clearly the objects near to him. The reason for Myopia is either the focal length of lens of eye is too short or the eyeball is very much elongated. In Myopia, light rays from a distant object are focused in front of the Retina



MYOPIA

Correction

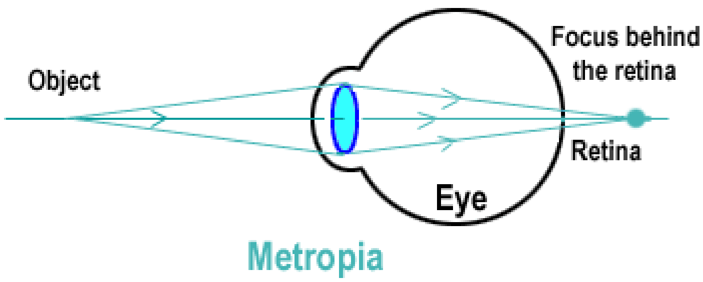
This defect can be corrected by using a concave lens of suitable focal length.



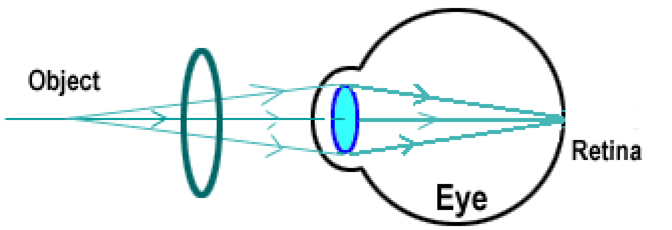
Myopia corrected by using concave lens

Long sightedness or hyper metropia

In **hyper metropia**, a person cannot see objects clearly which are near to him, but he can see clearly distant objects. The reason for **hyper metropia** is that either the focal length of the lens of eye is too long or the eyeball is too short. In **hyper metropia**, light rays from a near object are focused behind the Retina.



Correction
This defect can be corrected by using a convex lens of suitable focal length



Metropia corrected by using convex lens

Astigmatism
If the cornea or the surface of eye is not perfectly spherical, in this situation the eye has different focal points in different planes and an object is not focused clearly on the retina.

Correction
Astigmatism is corrected by using asymmetrical lenses which have different radii of curvature in different planes

Presbyopia
At old age, the eye lens loses its natural elasticity and ability to change its shape and the ciliary muscles weaken resulting in a lack of accommodation. This type of long sightedness is called "Presbyopia"

Correction
This defect can be corrected by using convex lens for long sighted person and concave lens for short sighted person.